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NEW TIME-VARYING SLIDING SURFACES FOR ROBUST VARIABLE STRUCTURE CONTROL SYSTEMS

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Most of sliding surfaces proposed so far for a variable structure control system(VSCS) have been determined independently of given initial conditions. The VSCS with these typical surfaces may be sensitive to paramter variations and extraneous disturbances during the reaching phase. To overcome this drawback, we propose a new time-varying sliding surface. The surface is initially designed to pass arbitrarily given initial conditions, and subsequently moved towards a predetermined sliding surface by rotating or/and shifting. The existence of sliding mode with the time-varying surface is proved, and moving procedures are presented as well as salient features. Using the proposed surface a low sensitivity system is obtained through shortening the reaching phase. Furthermore, the system robustness is almost guaranteed during whole intervals of control action by eliminating the reaching phase. To illustrate the advantages of the proposed method, a simple second-order linear system subjected to external disturbance is considered as preliminary example followed by a two-link manipulator.

Key Words: Variable Structure System, System Sensitivity, Robust Control, Time-Varying Sliding Surface, Robotic Manipulator.

1. INTRODUCTION

The problem of controlling uncertain dynamical systems subjected to extraneous disturbances has been studied for a long time. One deterministic approach to this problem is by means of a variable structure control system(VSCS). The VSCS is a special class of nonlinear control mechanism characterized by a discontinuous control action which changes the structure upon reaching a set of sliding surfaces (Utkin, 1978). The most important property of the VSCS is that the sliding motion of the state on the sliding surface is ensured. During the sliding motion, the system has invariance properties yielding motion which is independent of certain system variations and disturbances. The representative point of the system is constrained to move along a predetermined sliding surface (Itkis, 1976). Therefore, the design of the sliding surface completely determines the performance of the system.

Most of sliding surfaces proposed so far have been designed without consideration of given initial conditions. Using these surfaces, the sliding mode occurs only after the system reaches to the surfaces. Therefore, the VSCS may be sensitive to parameter variations and disturbances during the reaching phase. Furthermore, it is also known that the convergence to the surfaces may only be asymptotic, so that the benefits of the VSCS cannot be realized. When in the sliding mode, the system is completely insensitive to certain values of uncertain quantities if some invariance conditions are satisfied; that is, the disturbances and parameter variations will only affect the initial conditions of the sliding mode equations (Drazenovic, 1969, Spurgeon, 1991). Hence, the robustness of the VSCS can be improved by shortening the time required to attain the sliding mode, or may be guaranteed during whole intervals of control action by eliminating the reaching phase. One easy way to minimize the reaching phase is to employ the larger control input. Young et al.(1977) used the high-gain feedback to speed up the reaching phase. However, this may cause extreme sensitivity to unmodelled dynamics, and also higher chattering which is undesirable in physical system. On the other hand, Slotine and Sastry(1983) suggested a sliding surface in the state space in order to eliminate the reaching phase by imposing a constraint that the initial errors be zero in tracking control. However, this situation is not general but strictly special. Typically the initial conditions of actual system may be located arbitrarily.

Up to now, researches on this problem considerably rare while numerous works on the VSCS seriously point out the drawback. In this paper, to remove the drawback a new sliding surface adaptable to arbitrary initial conditions is proposed. The surface is initially designed to pass given initial conditions and subsequently moved towards a predetermined sliding surface by rotating or/and shifting. We call it as a time-varying sliding surface comparing with conventional ones, for instances, employed by Choi and Jayasuriya(1987), Hong and Wu(1989), and Spurgeon(1991). Using the proposed surface, the system sensitivity to uncertainties is remarkably lessened through shortening the reaching phase. Furthermore, the robustness of the VSCS is guaranteed during whole periods of control action by almost eliminating the reaching phase with trade-off against regulating or tracking time.

To recall the sensitivity of the VSCS during the reaching phase, we take a simple single-input linear system as an illustrative example in Section 2. In Section 3, we introduce the time-varying sliding surface for a second-order VSCS by dividing it into two types-the surface with time-varying

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slope, and with time-varying intercept. Moving procedures of each one are described in details as well as its salient features. And the existence of sliding mode with the time-varying surface is proved. In Section 4, we apply the surface to the control of a two-link manipulator to demonstrate some advantages of the proposed method. To our knowledge, these are novel results in the research community of the VSCS.

2. SENSITIVITY OF THE VSCS

Consider the system given by the set of (n) differential first-order equations.

$$\dot{x}(t)=A\ x(t)+B\ u(t)+D\ f(t) \qquad (1)$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$ and $f(t) \in \mathbf{R}^p$ are the state, control and disturbance vectors, respectively, and A, B and D are (n × n), (n × m) and (n × p) constant matrices. Define a set of sliding surfaces as

$$S=C\ x(t) \qquad (2)$$

where C is constant (m × n) matrix and det (CB) ≠ 0. Then, the state of the system during the sliding mode is constrained to the subspace defined by the equation

$$C\ x(t)=0 \qquad (3)$$

Differentiating (3) with respect to time and substituting from (1) yields following equivalent control u_{eq} in a unique manner.

$$u_{eq}=-(CB)^{-1}\ C\ (A\ x(t)+D\ f(t)) \qquad (4)$$

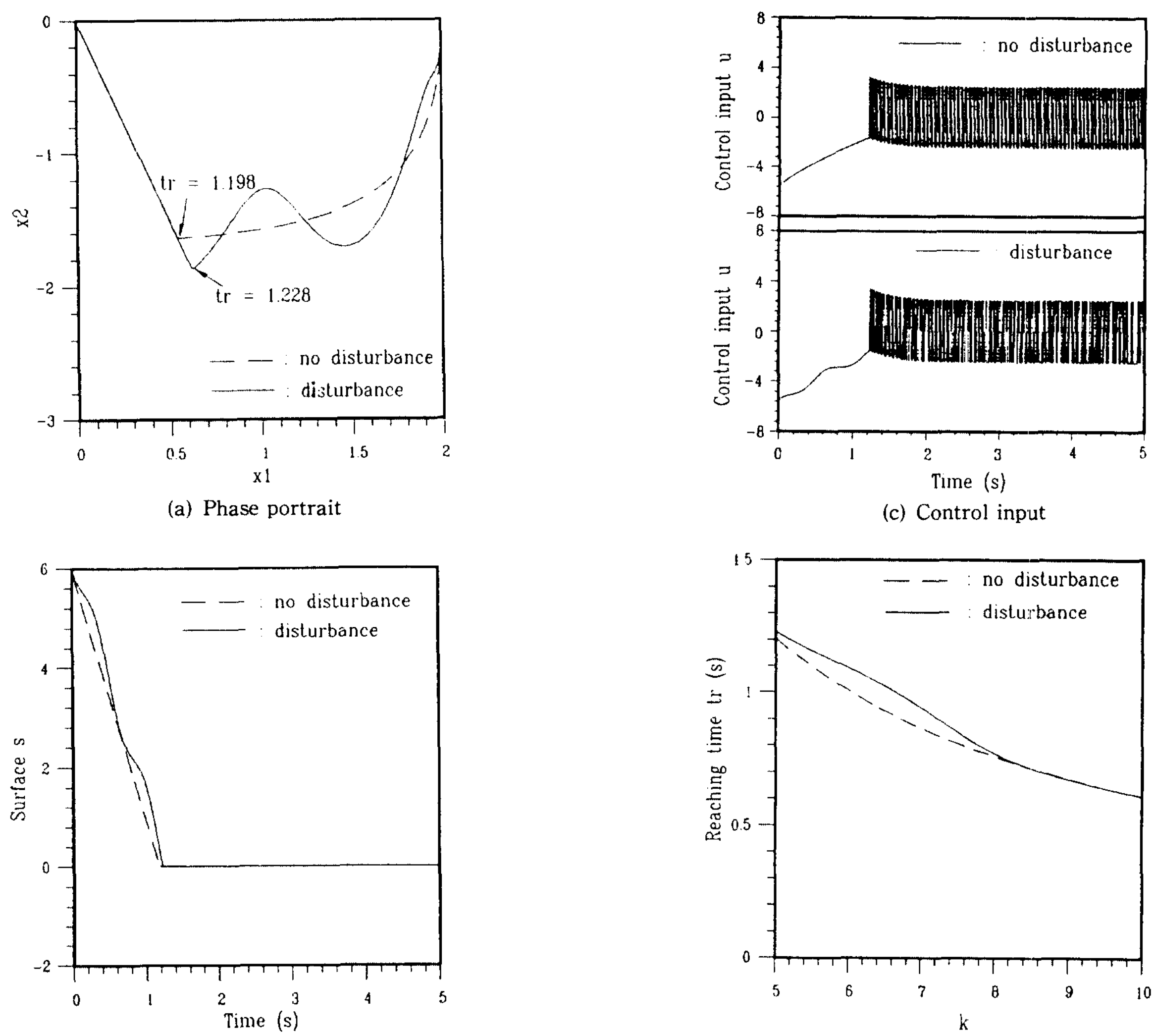
Thus, the resultant sliding mode equations are obtained as follows.

$$\dot{x}(t)=[I-B(CB)^{-1}C][A\ x(t)+D\ f(t)] \qquad (5)$$
$$C\ x(t)=0$$

where I is the unity matrix. It is seen from this equation that the disturbances $f(t)$, in general, act in equations of the sliding mode motion. The conditions for the sliding mode system (5) to be completely insensitive to the external disturbances $f(t)$ are well known (Drazenovic, 1969), i.e.

$$rank[B : D]=rank\ B \qquad (6)$$

where $[B\ :\ D]$ is a matrix composed of all the columns of B and D. Though if it is possible to satisfy some conditions dependent on matrices which define the control function and disturbance points to the system, the sliding mode motion will depend upon the disturbances through the given initial condi-



(a) Phase portrait (c) Control input

(b) Surface trajectory (d) Reaching time t_r vs. gain k

Fig. 1 Effect of the sinusoidal disturbance on the VSCS tions. Hence, if the preliminary part of the motion is shortened by a suitable choice of control function or other means, the time of disturbance influence can be remarkably decreased, so that the whole system exhibits a low sensitivity to distrubances.

To recall the sensitivity of the VSCS during the reaching phase, we consider typical second-order linear system described by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= a_1x_1(t) + a_2x_2(t) + bu(t) + d(t) \\ x(t_0) &= x_0 \end{aligned} \tag{7}$$

where a_1, a_2 and $b(\neq 0)$ are known constants. In Eq. (7), x_0 are initial conditions given at initial time t_0 and $d(t)$ is unknown but possibly bounded external disturbance as $|d(t)| \leq \hat{d}$. Defining a typical sliding surface (a line in this case)

$$s(x) = cx_1 + x_2, \ c > 0 \tag{8}$$

yields the discontinuous control in view of (4)

$$u(t) = -[a_1x_1 + (c + a_2)x_2 + k \ sgn(s(x))]/b \tag{9}$$

where k might be any positive number satisfying $k > \hat{d}$. Then the sliding condition for existence of sliding mode motion of the system (7)

$$s(x) \ \dot{s}(x) < 0, \ for \ x \in R^2 - s(x) \tag{10}$$

can be satisfied. In sliding motion $s(x)$ remains zero, and thus the resultant equation of the sliding mode becomes

$$s(x) = cx_1 + x_2 = 0 \tag{11}$$

It is obvious that the response of the system (11) depends only on the constant parameter c, hence providing the invariancy of the sliding mode with respect to the disturbance. The invariancy property is justified from the fulfillment of the invariance condition (6) for the canonical form of (7). The system (7) with the controller (9), however, may be sensitive to the disturbance when the system does not reach the sliding surface as shown in Fig. 1. For the simulation, the following values are employed : $a_1 = 3$, $a_2 = -1$, $b = 2$, $c = 3$, $k = 5$, $d(t) = 3\sin(3\pi t)$ and $(x_1(0), x_2(0)) = (2.0)$. In Fig. 1, t_r denotes the reaching time of the representative point taken from initial conditions to the surface. We clearly observe that after beginning of the sliding mode the specific sinusoidal distortions vanished. However, the imposed disturbance delays the starting moment of the sliding mode resulting in considerable change of the sliding mode initial conditions. It is also observed that the control input signal is affected by the disturbance during the reaching phase. One possibility of shortening the reaching time, thus lessening the disturbance influence is to increase the magnitude of the discontinuous control gain k as shown in Fig. 1(d). This, however, may cause extreme system sensitivity to unmodelled dynamics, actuator saturation and undesirable higher chattering as well. In the present work, we improve the robustness of the system without increasing the gain of k by introducing a new time-varying sliding surface.

3. TIME-VARYING SLIDING SURFACE

As mentioned in introduction, the basic philosophy of the time-varying sliding surface is that the surface is initially chosen to pass given arbitrary initial conditions, and we subsequently move the surface towards the predetermined sliding surface. The movement can be executed by rotating or/and shifting. Thus, we divide the time-varying surface into two types ; the surface with time-varying slope and with time-varying intercept. The movement for the former is associated with time-varying slope of the surface which belongs to a step function to be defined below. On the other hand, the movement for the latter is accomplished by employing time-varying intercept of the surface which also belongs to a step function.

Definition 1 : A function $\varphi : \mathbf{R} \rightarrow \mathbf{R}$ defined on $[a, b]$ is called a step function if there is a partition given by $a = v_1 < v_2 < \cdots\cdots < v_n = b$, such that φ is constant on each open subinterval (v_{k-1}, v_k), $1 < k \leq n$.

3.1 Surface with Time-Varying Slope

For the second-order system (7), let us define the sliding surface as

$$\begin{aligned} s_r(x(t), t) &= c_r x_1(t) + x_2(t) \\ s_r(x(t_0), t_0) &= s_{r0} = c_{r0} x_1(t_0) + x_2(t_0) \end{aligned} \tag{12}$$

We obviously see that the surface initially goes through given initial conditions $x(t_0)$ with the corresponding slope c_{r0}. In other words, the representative point(RP) initially lies on the surface s_{r0} as shown in Fig. 2(a). In this figure, s_p represents the predetermined sliding surface defined by $s_p = c_px_1(t) + x_2(t)$. Before describing a moving algorithm, we summarize the argument for existence of sliding mode in a following theorem.

Theorem 1 : If $c_r(t)$ in Eq. (12) is chosen to be a step function for $t \in [t_0, t_r]$ with terminal values of $c_r(t_0) = -x_2(t_0)/x_1(t_0)$ and $c_r(t_r) = c_p$, and $c_r(t)$ be a constant function when $t \in (t_r, \infty)$ with $c_r(t) = c_p$, the system (7) with controller (9) incorporating the sliding surface (12) satisfies sliding condition $s_r(x, t) \ \dot{s}_r(x, t) < 0$ almost everwhere.

Proof : From definition 1, there exists a partition $P = \{v_1, v_2, \cdots\cdots, v_n\}$, i.e., $t_0 = v_1 < v_2 < \cdots\cdots < v_n = t_r$ such that c_r(t) is constant on each open subinterval (v_{k-1}, v_k), $1 < k \leq n$. Trivially, P is a finite set. So we can prove that P is measurable and $m(P) = m_e(P) = 0$ (see Cohn, 1980), where m denotes Lebesgue measure and m_e stands for Lebesgue exterior measure. Therefore, since we chose $c_r(t)$ to be a step function on $[t_0, t_r]$, $\dot{c}_r(t) = 0$ for $t \in [t_0, t_r] - P$ and $\dot{c}_r(t) = 0$ for $t \in (t_r, \infty)$. Hence the control system (7) and (9) with the sliding surface (12) obviously satisfies sliding condition : when $t \in [t_0, t_r]$, $s_r(x, t) \ \dot{s}_r(x, t) < 0$ for $x \in \mathbf{R}^2 - s_r$, $t \in [t_0, t_r] - P$ and when $t \in (t_r, \infty)$, $s_r(x, t) \ \dot{s}_r(x, t) < 0$ for $x \in \mathbf{R}^2 - s_r$. This completes the proof.

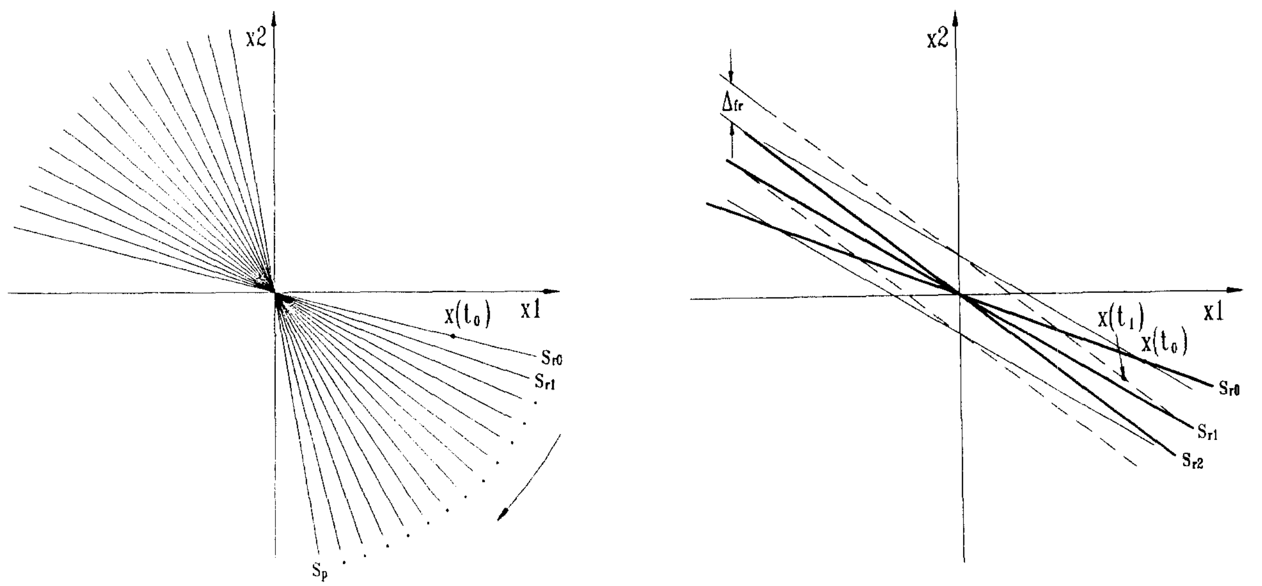
Now we can move the sliding surface s_{r0} to the s_p by employing time-varying slope $c_r(t)$ without violating the sliding condition almost everywhere. The moving algorithm proposed in this study may be outlined as follows.

Step 1.

We determine an appropriate constant Δ_r required to rotate the surface and define (refer to Fig. 2(b)) $\Delta_{fr} = \Delta_f + \Delta_r$, where Δ_f denotes the vicinity magnitude of the surface due to nonidealities such as delay, hysteresis and etc. The value of Δ_r plays a crucial role for improving the the robustness of the system. The smaller value of Δ_r, the shorter reaching time, hence resulting in low system sensitivity(see step 4). If Δ_r approaches to zero, the RP may cross the sliding surface. In other words, the reaching phase may be almost eliminated resulting in the enhancement of the system robustness with trade-off against regulating or tracking time.

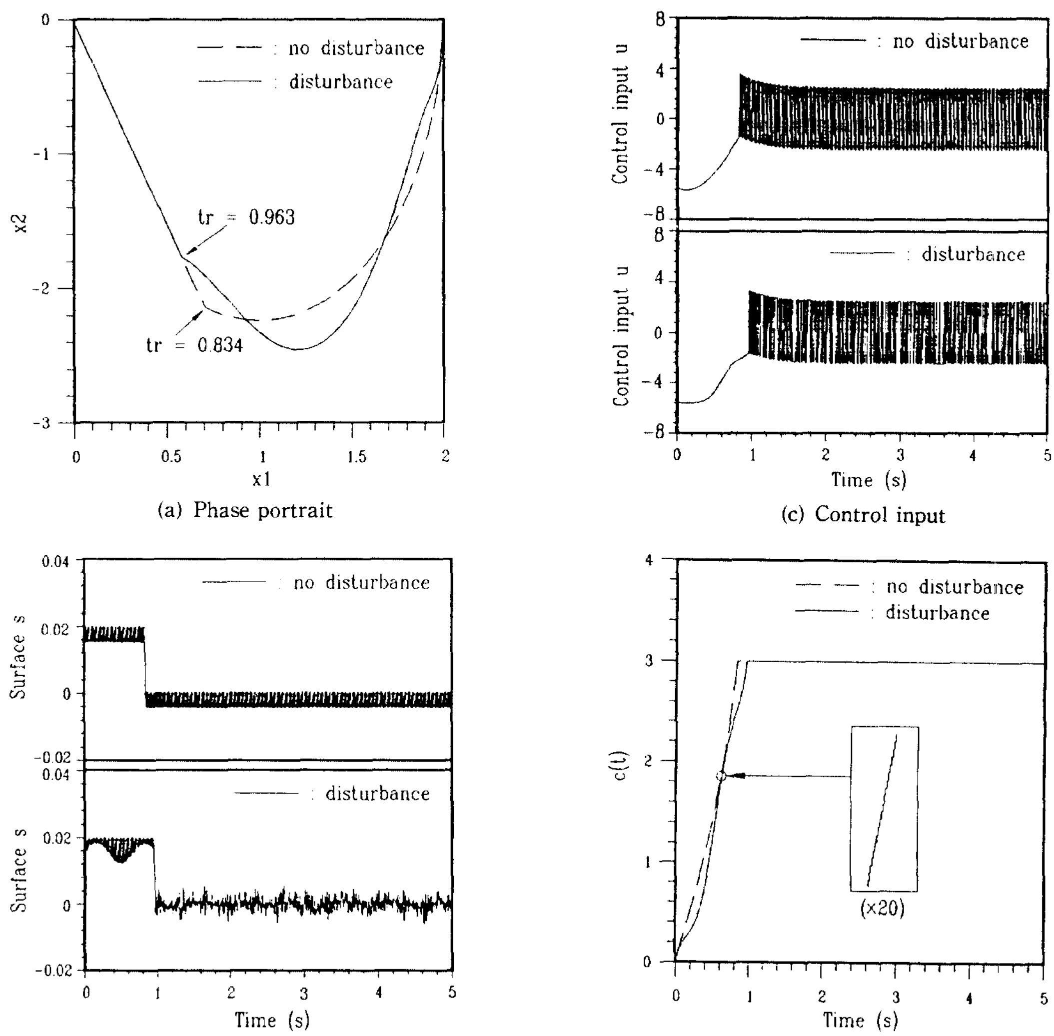
Step 2.

We calculate the initial slope c_{r0} satisfying the equation



(a) Configuration (b) Mechanism

Fig. 2 Surface with time-varying slope



(a) Phase portrait (c) Control input

(b) Surface trajectory (d) Slope variation

Fig. 3 Control responses with the sliding surface (12) (dwelling time=0.001 sec)

$s_{r0}=0$ according to given initial conditions $x(t_0)$; $c_{r0}=-x_2(t_0)/x_1(t_0)$.

Step 3.

The rotating direction is determined from the values of c_{r0} and c_p, i.e., if $c_{r0}<c_p$; clockwise(CW), and if $c_{r0}>c_p$; counter-clockwise(CCW).

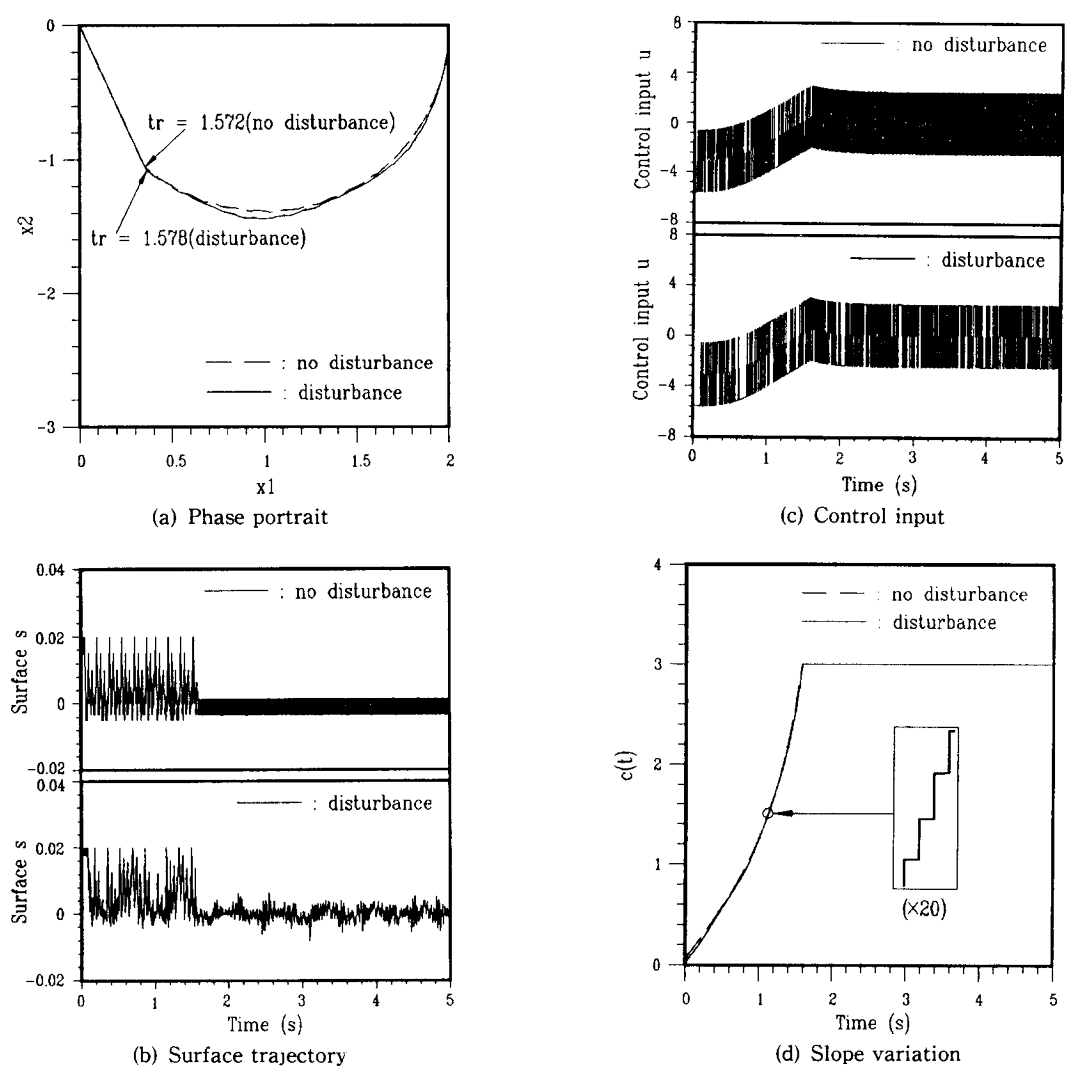
Step 4.

We instantaneously rotate the s_{r0} to s_{r1} which has the slope c_{r1} obtained by solving the equation $|c_{r1}x_1(t_0)+x_2(t_0)|=\Delta_{fr}$. The larger value of two solutions of c_{r1} is chosen as the slope for clockwise, and the other for counter-clockwise. The surface s_{r1} stays for a finite time (we call it as the dwelling time ($\Delta\tau$) of the surface) before moving to the next surface s_{r2} whose slope c_{r2} is obtained by solving the equation $|c_{r2}x_1(t_1)+x_2(t_1)|=\Delta_{fr}$, where $t_1=t_0+\Delta\tau$. We know that the dwelling time $\Delta\tau$ also plays a crucial role as the Δ_r for the system performance. The shorter dwelling time $\Delta\tau$, the shorter reaching time t_r. If the $\Delta\tau$ is chosen to be long, the RP may cross the sliding surface. Then the control input signal which has opposite sign is activated to drive the RP to the opposite direction resulting in sluggish motion. However, with trade-off against regulating or tracking time the system robustness may be guaranteed during whole intervals of control action. The rotating is continuously performed in a same manner until following step is checked.

Step 5.

We stop the rotating under following condition, i.e., if $c_{rn}>c_p$, then fix $c(t)=c_p$: CW, and if $c_{rn}<c_p$, then fix $c(t)=c_p$: CCW. The slope c_{rn} is obtained by solving the equation $|c_{rn}x_1(t_{n-1})+x_2(t_{n-1})|=\Delta_{fr}$, where $t_{n-1}=t_0+(n-1)\Delta\tau$.

This algorithm is now applied to the system (7). Fig. 3 shows control responses obtained using the controller (9) in which the surface (8) is replaced by the proposed surface (12). Same values of a_1, a_2, b, k, $d(t)$ and $x(t_0)$ as those used for Fig. 1 are employed. The values of c_p, Δ_f and Δ_r are chosen as 3, 0.01, and 0.01, respectively, and the dwelling time $\Delta\tau$ is chosen to be 0.001 second. From the phase portrait, we clearly know that the reaching time t_r in the presence of the disturbance is significantly reduced by employing the surface (12) comparing with conventional one (the surface defined by the Eq. (8)). This improvement of the system robustness without increasing the gain k, hence undesirable chattering will furnish lots more benefits in practice. It is observed from the surface



(a) Phase portrait

(b) Surface trajectory

(c) Control input

(d) Slope variation

Fig. 4 Control responses with the sliding surface (12) (dwelling time=0.01 sec) trajectory that the RP never crosses the surface during the reaching phase. The chattering magnitude of the surface during the reaching phase is due to the imposed dwelling time $\Delta\tau$. The longer $\Delta\tau$, the larger magnitude. It is noted that the chattering in this phase does not cause discontinuities of the control signals as shown in Fig. 3(c). From the magnification (20 times) of $c(t)$ in Fig. 3(d), we observe that the time-varying slope $c(t)$ indeed falls into the step function.

Fig. 4 presents control responses of the system (7) obtained imposing the dwelling time $\Delta\tau=0.01$ second instead of 0.001 second in Fig. 3. It is clearly observed from the phase portrait that the sinusoidal distortion due to the disturbance is almost vanished during whole periods of control action with trade-off against the reaching time t_r. From the surface trajectories, we observe that the RP crosses the sliding surface from the beginning, and thus, improving the system robustness. We may say that there is no reaching phase in this case except changing time of the slope. It is noted that the imposing of longer dwelling time might be very useful to the system subjected to high magnitudes of uncertainties, which can possibly cause a catastrophe all of a sudden.

One can naturally ask how about the initial conditions are located in the unstable zone, i.e., the first and third quadrants in the above example. If we define the sliding surface to go through the initial conditions and the origin as well, the surface itself is unstable. Therefore, it is no doubt that the RP on the sliding surface goes away from the origin until it arrives to stable zone. From the mathematical point of view, though it is possible to drive the RP to the origin in a finite time by employing the surface (12), this may cause much longer reaching time t_r than the conventional one. To avoid this problem we propose following sliding surface.

3.2 Surface with Time-Varying Intercept

We define the sliding surface for the system (7) as

$$s_s(x(t),\ t)=c_p x_1(t)+x_2(t)-\alpha(t)$$
$$s_s(x(t_0),\ t_0)=s_{s0}=c_p x_1(t_0)+x_2(t_0)-\alpha_0 \tag{13}$$

where c_p is the slope of the predetermined sliding surface s_p. and $\alpha(t)$ is the time-varying intercept of the x_2 axis. The surface initially goes through given initial conditions with appropriate initial intercept α_0 as shown in Fig. 5(a). Similar to the surface (12), we obtain following theorem regarding to the existence of sliding mode with the sliding surface (13).

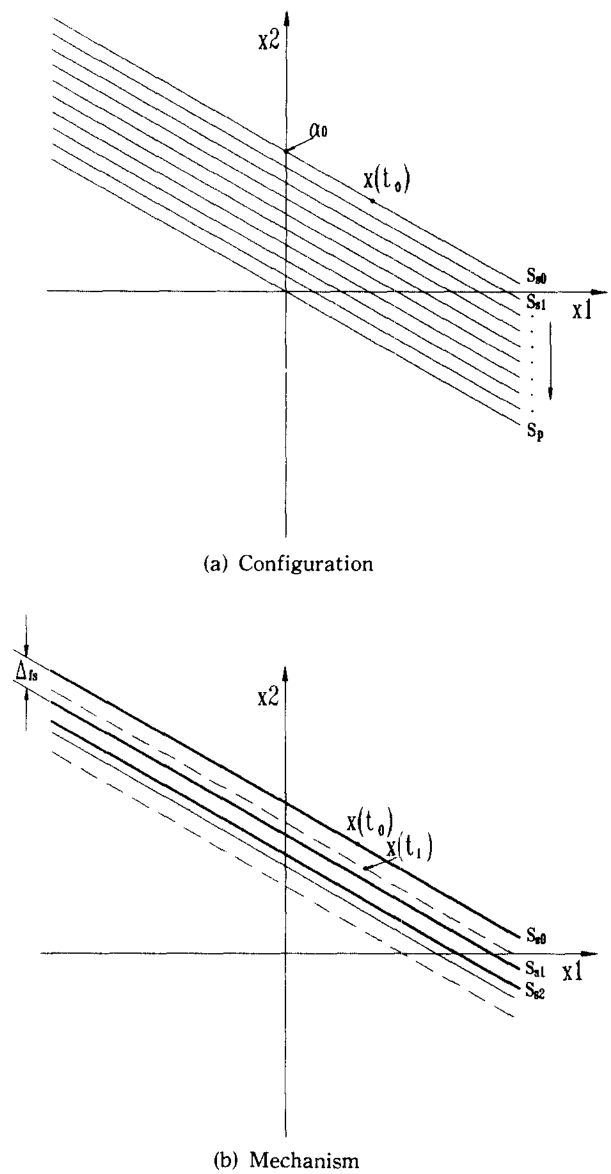


Fig. 5 Surface with time-varying intercept

Theorem 2 : If $\alpha(t)$ in Eq. (13) is chosen to be a step function for $t\in[t_0,\ t_r]$ with terminal values of $\alpha(t_0)=x_2(t_0)+c_p x_1(t_0)$ and $\alpha(t_r)=0$, and $\alpha(t)$ be a constant function when $t\in(t_r,\ \infty)$ with $\alpha(t)=0$, the system (7) with controller (9) incorporating the sliding surface (13) satisfies sliding condition $s_s(x,\ t)\ \dot{s}_s(x,\ t)<0$ almost everwhere.

The proof can be easily completed similar to the proof of theorem 1. The movement is performed by calculating updated intercept $\alpha(t)$ until the surface s_{s0} becomes to the s_p. The algorithm to move the s_{s0} to the s_p is outlined as follows.

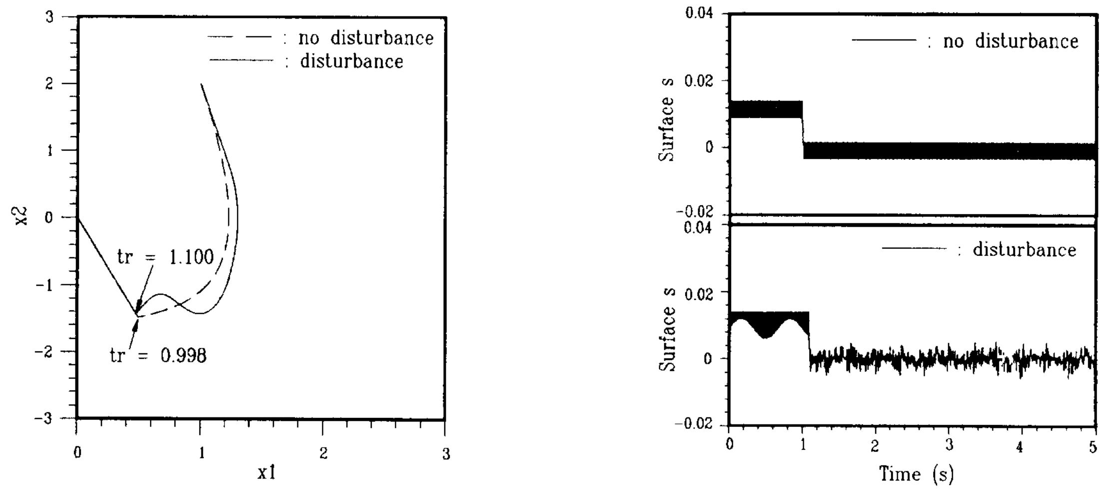
Step 1.
We determine an appropriate constant Δ_s required to shift the surface and define (refer to Fig. 5(*b*)) $\Delta_{fs}=\Delta_f+\Delta_s$.

Step 2.
We calculate the initial intercept α_0 satisfying the equation $s_{s0}=0$ according to given initial conditions $x(t_0)$; $\alpha_0=c_p x_1(t_0)+x_2(t_0)$.

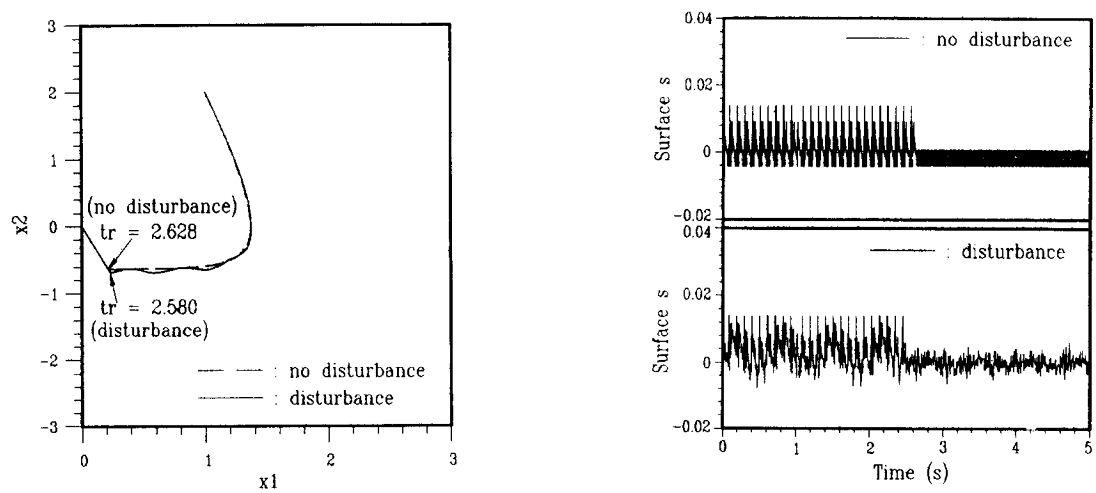
Step 3.
The shifting direction is determined from the value of α_0, i.e., if $\alpha_0>0$; upward, and if $\alpha_0<0$; downward.

Step 4.
The surface s_{s0} is immediately shifted to the s_{s1} which has the intercept of α_1 obtained by solving the equation $|\ c_p x_1(t_0)+x_2(t_0)-\alpha_1\ |=\Delta_{fs}$. The larger value of two solutions of α_1 is chosen as the intercept for upward, and the other for downward. The surface s_{s1} stays for a finite time ($\Delta\tau$) before shifting to the next surface s_{s2} whose intercept α_2 is obtained by solving the equation $|\ c_p x_1(t_1)+x_2(t_1)-\alpha_2\ |=\Delta_{fs}$, where $t_1=t_0+\Delta\tau$. The shifting is continuously undertaken in a same manner until following step is checked.

Step 5.
We stop the shifting under following condition, i.e., if $\alpha_n>0$, then fix $\alpha(t)=0$; upward, and if $\alpha_n<0$, then fix $\alpha(t)=0$; downward. The intercept α_n is obtained by solving the equation $|\ c_p x_1(t_{n-1})+x_2(t_{n-1})-\alpha_n\ |=\Delta_{fs}$, where $t_{n-1}=t_0+(n-1)\Delta\tau$.

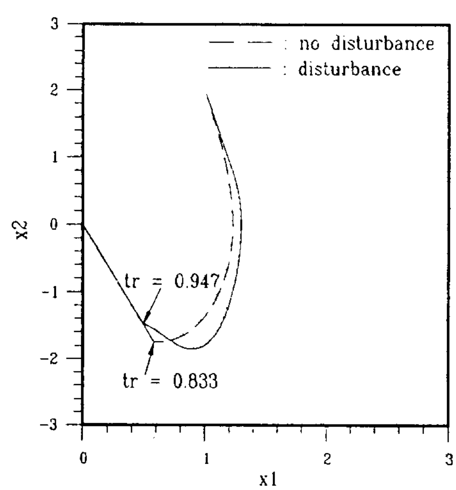


(a) Dwelling time = 0.001 sec

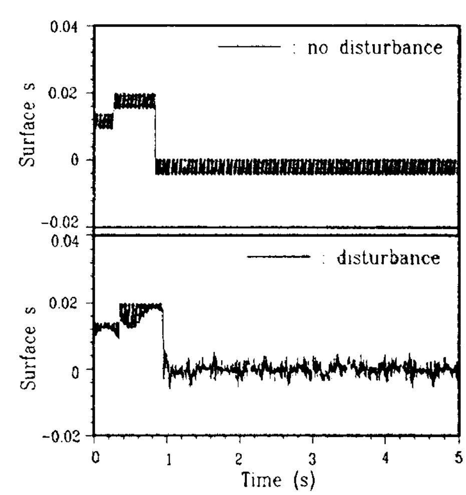


(b) Dwelling time = 0.007 sec

Fig. 6 Control responses with the sliding surface (13)



(a) Phase portrait



(b) Surface trajectory

Fig. 7 Control responses with the sliding surface (14)

Fig. 6 shows control responses of the system (7) obtained using the controller (9) in which the proposed surface (13) is employed instead of the surface (8). Same values of a_1, a_2, b, k, $d(t)$ and Δ_t as those used for Fig. 3 are employed, and initial conditions of $(x_1(t_0), x_2(t_0)) = (1, 2)$ are imposed. The value of Δ_s is chosen as 0.004. We clearly know that the surface trajectory is absolutely different from conventional one. Furthermore, using the proposed surface the robustness of the system can be obtained in a different sense by imposing longer dwelling time $\Delta\tau$ as shown in Fig. 6(b). The sinusoidal distortion due to the disturbance is almost vanished during whole intervals of control action.

From the intuition, we may combine the surface (12) and (13) to get better result in the sense of reducing the reaching time as well as increasing the robustness. For instance, if the initial conditions are located in the unstable zone, the surface (13) is used until the RP moves to the stable zone, and subsequently the surface (12) is employed throughout. Consequently, we may define the time-varying sliding surface as

$$
\begin{aligned}
s_m(x(t),\ t) &= c(t)x_1(t) + x_2(t) - \alpha(t) \\
s_m(x(t_0),\ t_0) &= c(t_0)x_1(t_0) + x_2(t_0) - \alpha(t_0)
\end{aligned}
\tag{14}
$$

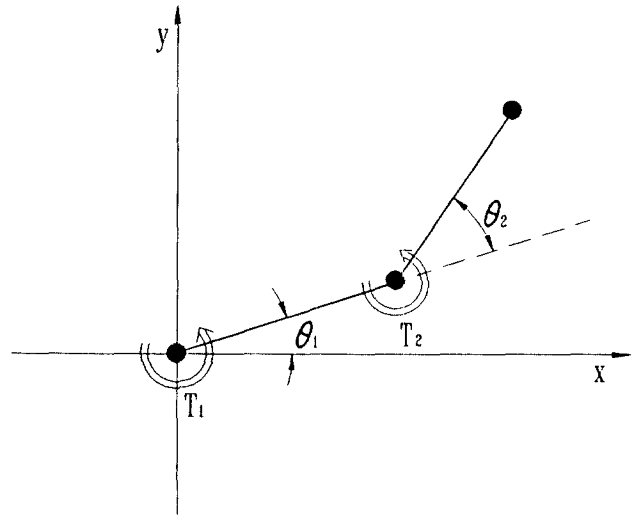
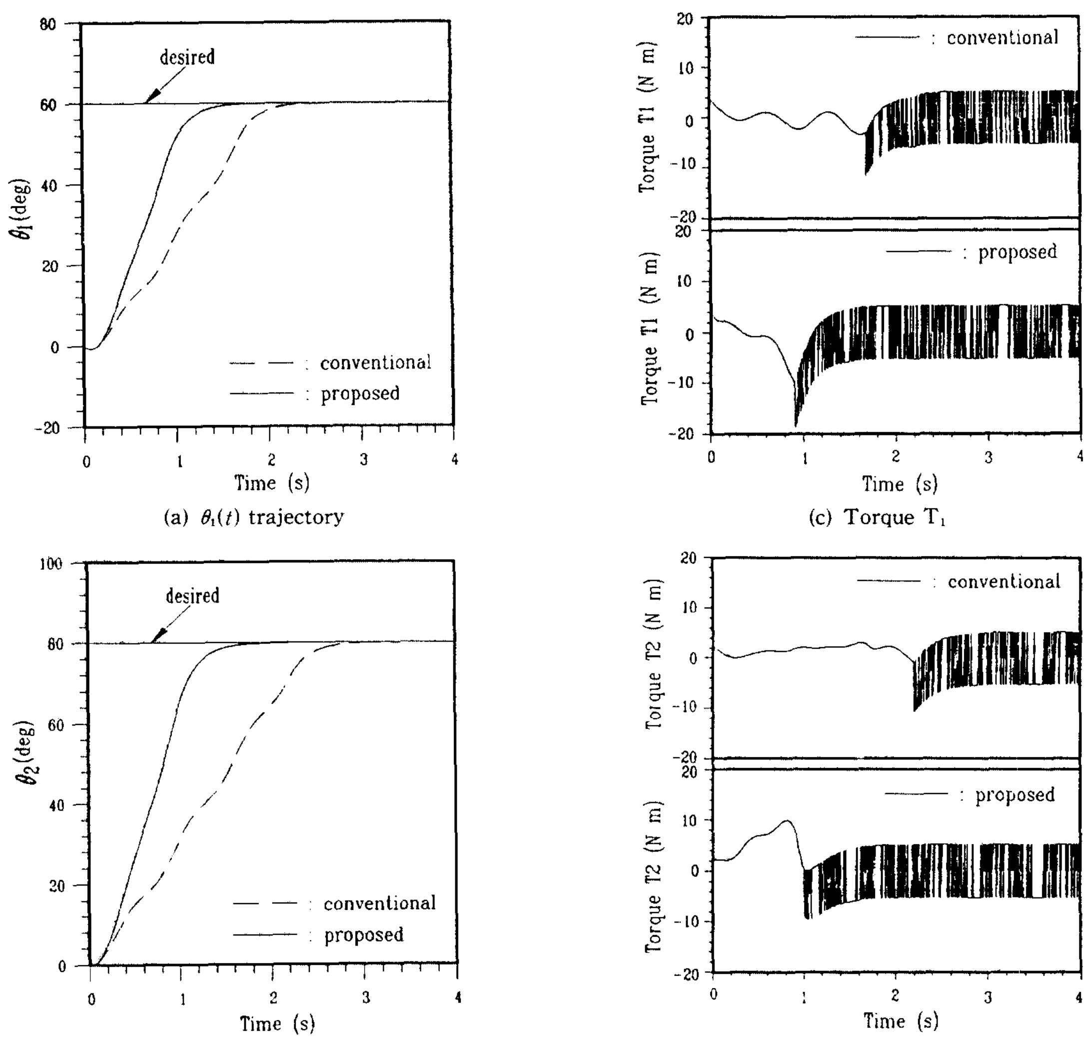


Fig. 8 A two-link manipulator



(a) $\theta_1(t)$ trajectory

(b) $\theta_2(t)$ trajectory

(c) Torque T_1

(d) Torque T_2

Fig. 9 Angle trajectories and control inputs of the manipulator

The intercept $\alpha(t)=0$ for the surface (12) and the slope $c(t)=c_p$ for the surface (13). Fig. 7 presents control responses of the system (7) obtained using the surface (14). All parameters are same as those for Fig. 6(a). We clearly observe that the reaching time t_r is shortened by comparing with one shown in Fig. 6(a). From the surface trajectory, we easily know that the surface (13) with $\Delta_s=0.004$ is executed first followed by the surface (12) with $\Delta_r=0.01$. In general, we start the movement (shifting or rotating) according to the location of initial conditions. In the subsequent section, we apply the surface (14) to the control of a two-link manipulator.

4. APPLICATION TO A TWO-LINK MANIPULATOR

To show the effectiveness of the proposed method we apply the time-varying surface (14) to the control of a two-link manipulator shown in Fig. 8. By assuming normalized unit mass and unit length of the arm the dynamic equations are obtained as follows(Slotine and Sastry, 1983).

$$\ddot{\theta}_1=[2/3\sin\theta_2\dot{\theta}(2\dot{\theta}_1+\dot{\theta}_2)+(2/3+\cos\theta_2)\sin\theta_2\dot{\theta}_1^2+T_1]/(16/9-\cos^2\theta_2)+d_1(t)$$

$$\ddot{\theta}_2=[-(2/3+\cos\theta_2)\sin\theta_2\dot{\theta}_2(2\dot{\theta}_1+\dot{\theta}_2)-2(5/3+\cos\theta_2)\sin\theta_2\dot{\theta}_1^2+T_2]/(16/9-\cos^2\theta_2)+d_2(t) \quad (15)$$

where $d_1(t)$ and $d_2(t)$ are unknown but bounded external torque disturbances. The control problem is to regulate the angles $(\theta_1(t),\ \theta_2(t))$ to a desired configuration $(\theta_{d1},\ \theta_{d2})$. Thus the control torques T_1 and T_2 should be determined to undergo that the trajectory error is to be zero asymptotically for any given initial conditions. Accordingly, in view of (14) we define the time-varying sliding surfaces as

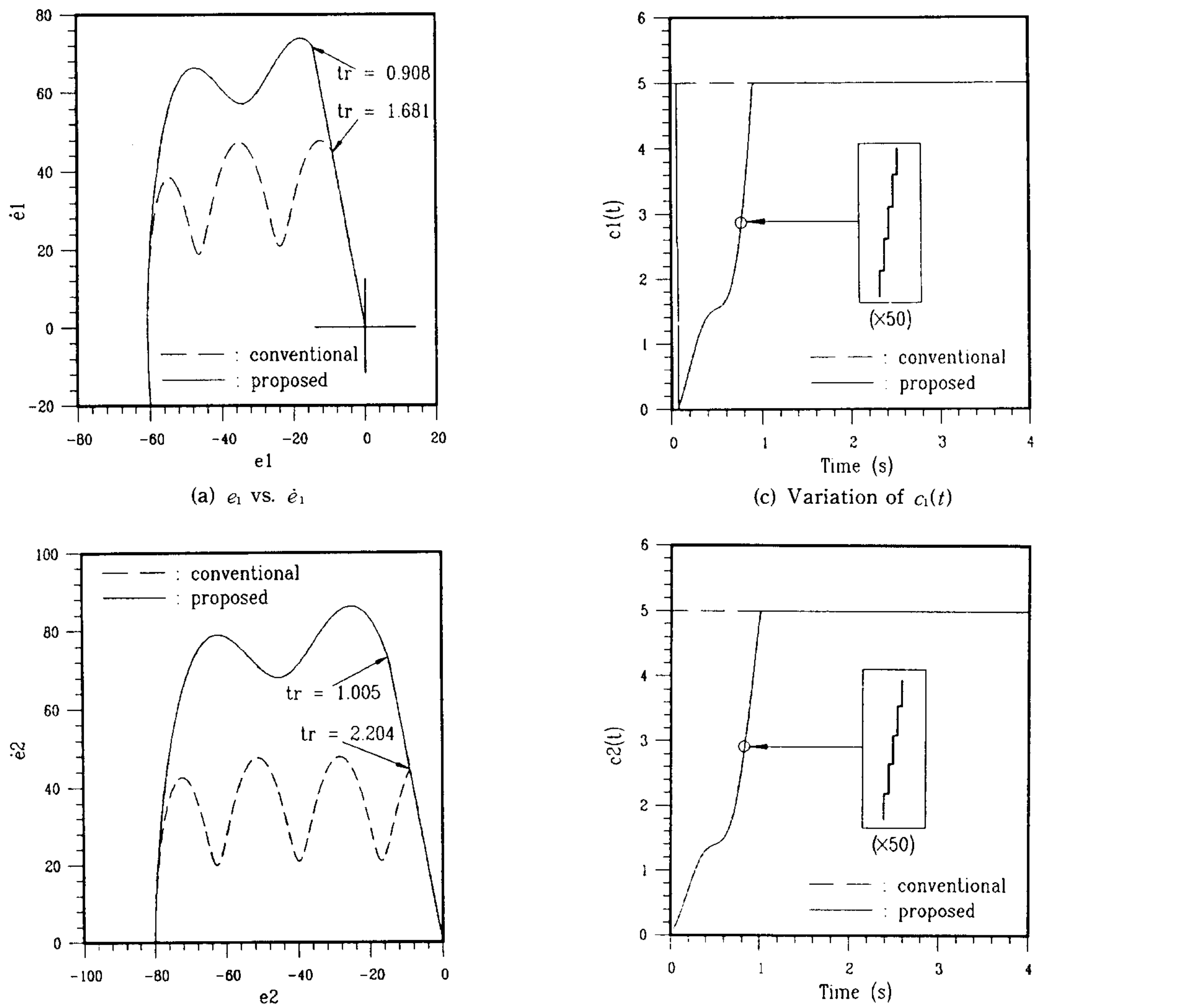
$$s_{mi}(t)=c_i(t)(\theta_i(t)-\theta_{di})+(\dot{\theta}_i(t)-\dot{\theta}_{di})-\alpha_i(t)$$
$$s_{mi}(t_0)=c_i(t_0)(\theta_i(t_0)-\theta_{di})+(\theta_i(t_0)-\dot{\theta}_{di})-\alpha_i(t_0),\ i=1,\ 2 \quad (16)$$

Though the control problem is a multi-input case, it is treated as m single-input problems; the i-th sliding surface s_{mi} depends only upon $\theta_i(t)$. Hence, from the concept of equivalent control given in Eq. (4) the discontinuous control laws to satisfy the sliding condition

$$s_{mi}(t)\dot{s}_{mi}(t)<0 \quad (17)$$

can be obtained as follows.

$$T_1=-2/3\sin\theta_2\dot{\theta}_2(2\dot{\theta}_1+\dot{\theta}_2)-(2/3+\cos\theta_2)\sin\theta_2\dot{\theta}_1^2+(16/9-\cos^2\theta_2)(-c_1(t)\dot{\theta}_1-k_1\mathrm{sgn}(s_{mi}(t)))$$
$$T_2=(2/3+\cos\theta_2)\sin\theta_2\dot{\theta}_2(2\dot{\theta}_1+\dot{\theta}_2)+2(5/3+\cos\theta_2)$$



(a) e_1 vs. $\dot{e}_1$ (c) Variation of $c_1(t)$

(b) e_2 vs. $\dot{e}_2$ (d) Variation of $c_2(t)$

Fig. 10 Phase portraits and variations of the slopes

$$\sin\theta_2\dot{\theta}_1^{\,2}+(16/9-\cos^2\theta_2)(-c_2(t)\dot{\theta}_2-k_2\mathrm{sgn}(s_{m2}(t)))\tag{18}$$

For the simulation, following numerical values are employed ; $\Delta_f=0.01$, $\Delta_r=0.008$, $\Delta_s=0.005$, $\Delta\tau=0.001$ second, $k_1=k_2=3.0$, $c_{P1}=c_{P2}=5.0$, $d_1(t)=d_2(t)=2.5\sin(3\pi t)$, $(\theta_1(0),\ \dot{\theta}_1(0),\ \theta_2(0),\ \dot{\theta}_2(0))=(0°,\ -20$ deg/sec, $0°$, odeg/sec) and $(\theta_{d1},\ \theta_{d2})=(60^0,\ 80^0)$. Fig. 9 presents the angle trajectory of $\theta_i(t)$ and the corresponding control torque of T_i. In this figure, the conventional sliding surface is defined by

$$s_i(t)=c_i(\theta_i(t)-\theta_{di})+(\dot{\theta}_i(t)-\dot{\theta}_{di}),\quad i=1,\ 2\tag{19}$$

where the value of constant slope c_i is chosen as 5.0 for i=1, 2. It is noted that the surfaces (19) are exactly same as ones used by Slotine and Sastry(1983). We clearly observe that the proposed method considerably improves the robustness of the system (15) by shortening the reaching phase without increasing the undesirable chattering magnitude of control torques. The sinusoidal distortions of the trajectories due to the disturbances are also remarkably reduced. It is noted that we can approximate the discontinuous control torques by continuous ones inside the boundary layer. Fig. 10 shows corresponding phase portraits in the error space and the variation of the slope $c_i(t)$. The error e_1 and e_2 is defined by $(\theta_1(t)-\theta_{d1})$ and $(\theta_2(t)-\theta_{d2})$, respectively. From the variation of the $c_1(t)$, we know that the surface $s_{m1}(t)$ is firstly shifted and after rotated until the $c_1(t)$ becomes equal to $c_{P1}(=5)$. On the other hand, the movement of the surface $s_{m2}(t)$ is executed by only rotating through whole moving intervals. The execution of the movement depends upon the location of given initial conditions. The time-varying sliding surface $s_{mi}(t)$ in the multi-input system is treated as the single-input case ; the i-th sliding surface moves independently. Without lose of generality, the proposed surface could be extended to path control problem. Simulation results presented in this work are quite self-explanatory justifying that the proposed method is very effective for improving the robustness of the system subjected to disturbances.

5. CONCLUSIONS

The new type of time-varying sliding surface has been proposed to improve the robustness of the VSCS. The surface was designed first to pass given initial conditions and subsequently move towards the predetermined sliding surface by rotating or/and shifting. Employing the proposed surface, it was possible to remarkably lessen the system sensitivity to extraneous distrubances by means of shortening the reaching phase without increasing undesirable chattering of the control input signals. Furthermore, reaching phase was almost eliminated by increasing the dwelling time of the surface, hence guaranteeing the system robustness during whole intervals of control action. It has been shown that the proposed method could be applied to both single-input and multi-input systems. In multi-input systems, each sliding surface moves independently according to given initial conditions. However, certainly the present work is only first step in developing the proposed method for general classes of linear and nonlinear systems. For instance, a logical method to determine the values of $c(t)$ and $\alpha(t)$ for higher-order systems is yet to be studied.

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